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Understanding Accelerated Life-Testing Analysis

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Summary & Purpose

Accelerated tests are becoming increasingly popular in today's industry due to the need for obtaining life data quickly. Life testing of products under higher stress levels without introducing additional failure modes can provide significant savings of both time and money. Correct analysis of data gathered via such accelerated life testing will yield parameters and other information for the product's life under use stress conditions.

This is a brief introductory tutorial on this subject. Its main purpose is to introduce the participant to some of the basic theories and methodologies of accelerated life testing data analysis.

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Mr. Vassiliou directs and coordinates ReliaSoft's R&D efforts to deliver state of the art software tools for applying reliability engineering concepts and methodologies. He is the original architect of ReliaSoft's Weibull++, a renowned expert and lecturer on Reliability Engineering and ReliaSoft's founder. He is currently spearheading the development of new technologically advanced products and services. In addition, he also consults, trains and lectures on reliability engineering topics to Fortune 1000 companies worldwide. Mr. Vassiliou holds an MS degree in Reliability Engineering from the University of Arizona.

Mr. Mettas is the Senior research scientist at ReliaSoft Corporation. He fills a critical role in the advancement of ReliaSoft's theoretical research efforts and formulations in the subjects of Life Data Analysis, Accelerated Life Testing, and System Reliability and Maintainability. He has played a key role in the development of ReliaSoft's software including Weibull++, ALTA, and BlockSim, and has published numerous papers on various reliability methods. Mr. Mettas holds an MS degree in Reliability Engineering from the University of Arizona.

Table of Contents

1. INTRODUCTION

Traditional "Life Data Analysis" involves analyzing times-to-failure data (of a product, system or component) obtained under "normal" operating conditions in order to quantify the life characteristics of the product, system or component. In many situations, and for many reasons, such life data (or times-to-failure data) is very difficult, if not impossible, to obtain. The reasons for this difficulty can include the long life times of today's products, the small time period between design and release, and the challenge of testing products that are used continuously under normal conditions. Given this difficulty, and the need to observe failures of products to better understand their failure modes and their life characteristics, reliability practitioners have attempted to devise methods to force these products to fail more quickly than they would under normal use conditions. In other words, they have attempted to accelerate their failures. Over the years, the term "*Accelerated Life Testing*" has been used to describe all such practices.

A variety of methods, which serve different purposes, have been termed "*Accelerated Life Testing.*" As we use the term in this tutorial, "*Accelerated Life Testing*" involves acceleration of failures with the single purpose of the "**quantification of the life characteristics of the product at normal use conditions.**" This tutorial is solely concerned with this type of accelerated life testing. To avoid confusion, the following section describes different types of tests that have been called "accelerated tests" and distinguishes between those that are addressed in this tutorial and those that are not.

2. TYPES OF ACCELERATED TESTS

Each type of test that has been called an accelerated test provides different information about the product and its failure mechanisms. Generally, accelerated tests can be divided into three types: *Qualitative Tests* (Torture Tests or Shake and Bake Tests), *ESS and Burn-in* and finally *Quantitative Accelerated Life Tests*. This tutorial only addresses and quantifies some models and procedures associated with the last type, ***Quantitative Accelerated Life Tests***.

2.1 *Qualitative Tests*

Qualitative Tests are tests that yield failure information (or failure modes) only. They have been referred to by many names including:

- Elephant Tests
- Torture Tests
- HALT (Highly Accelerated Life Testing)
- Shake & Bake Tests

Qualitative tests are performed on small samples with the specimens subjected to a single severe level of stress, to a number of stresses, or to a time-varying stress (i.e., stress cycling, cold to hot, etc.). If the specimen survives, it passes the test. Otherwise, appropriate actions will be taken to improve the product's design in order to eliminate the cause(s) of failure. Qualitative tests are used primarily to reveal probable failure modes. However, if not designed properly, they may cause the product to fail due to modes that would have never been encountered in real life. A good qualitative test is one that quickly reveals those failure modes that will occur during the life of the product under normal use conditions. In general, qualitative tests ***are not designed to yield life data that can be used in subsequent analysis or for "Accelerated Life Test Analysis."*** In general, qualitative tests do not quantify the life (or reliability) characteristics of the product under normal use conditions.

2.1.1 *Benefits and Drawbacks of Qualitative Tests:*

Benefit: Increase reliability by revealing probable failure modes.

Unanswered question: What is the reliability of the product at normal use conditions?

2.2 *ESS and Burn-In*

The second type of accelerated test consists of ESS and Burn-in testing. ESS, Environmental Stress Screening, is a process involving the application of environmental stimuli to products (usually electronic or electromechanical products) on an accelerated basis. The stimuli in an ESS test can include thermal cycling, random vibration, electrical stresses, etc. The goal of ESS is to expose, identify and eliminate latent defects which cannot be detected by visual inspection or electrical testing but which will cause failures in the field. ESS is performed on the entire population and does not involve sampling.

Burn-in can be regarded as a special case of ESS. According to MIL-STD-883C, Burn-in is a test performed for the purpose of screening or eliminating marginal devices. Marginal devices are those with inherent defects or defects resulting from manufacturing aberrations which cause time- and stress-dependent failures. As with ESS, Burn-in is performed on the entire population. Readers interested in the subject of ESS and Burn-in are encouraged to refer to Kececioglu & Sun on ESS [3] and Burn-in [4].

2.3 *Quantitative Accelerated Life Tests*

Quantitative Accelerated Life Testing, unlike the qualitative testing methods (i.e., Torture Tests, Burn-in, etc.) described previously, consists of quantitative tests designed to quantify the life characteristics of the product, component or system under normal use conditions, and thereby provide "Reliability Information." Reliability information can include the determination of the probability of failure of the product under use conditions, mean life under use conditions, and projected returns and warranty costs. It can also be used to assist in the performance of risk assessments, design comparisons, etc.

Accelerated Life Testing can take the form of "Usage Rate Acceleration" or "Overstress Acceleration." Both Accelerated Life Test methods are described next. Because "Usage Rate Acceleration" test data can be analyzed with typical life data analysis methods, the Overstress Acceleration method is the testing method relevant to this Tutorial.

For all life tests, some time-to-failure information for the product is required since the failure of the product is the event we want to understand. In other words, if we wish to understand, measure, and predict any event, we must observe the event!

Most products, components or systems are expected to perform their functions successfully for long periods of time, such as years. Obviously, for a company to remain competitive, the time required to obtain times-to-failure data must be considerably less than the expected life of the product. Two methods of acceleration, "Usage Rate Acceleration" and "Overstress Acceleration," have been devised to obtain times-to-failure data at an accelerated pace. For products that do not operate continuously, one can accelerate the time it takes to induce failures by continuously testing these products. This is called "Usage Rate Acceleration." For products for which "Usage Rate Acceleration" is impractical, one can apply stress(es) at levels that exceed the levels that a product will encounter under normal use conditions and use the times-to-failure data obtained in this manner to extrapolate to use conditions. This is called "Overstress Acceleration."

2.3.1 *Usage Rate Acceleration*

For products that do not operate continuously under normal conditions, if the test units are operated continuously, failures are encountered earlier than if the units were tested at normal usage. For example, a microwave oven operates for small periods of time every day. One can accelerate a test on microwave ovens by operating them more frequently until failure. The same could be said of washers. If we assume an average washer use of 6 hours a week, one could conceivably reduce the testing time 28-fold by testing these washers continuously. Data obtained through usage acceleration can be analyzed with the same methods used to analyze regular times-to-failure data. The limitation of "Usage Rate Acceleration" arises when products, such as computer servers and peripherals, maintain a very high or even continuous usage. In such cases, usage acceleration, even though desirable, is not a feasible alternative. In these cases the practitioner must stimulate the product to fail, usually through the application of stress(es). This method of accelerated life testing is called "Overstress Acceleration" and is described next.

2.3.2 *Overstress Acceleration*

For products with very high or continuous usage, the accelerated life-testing practitioner must stimulate the product to fail in a life test. This is accomplished by applying stress(es) that exceed the stress(es) that a product will encounter under normal use conditions. The times-to-failure data obtained under these conditions are then used to extrapolate to use conditions. Accelerated life tests can be performed at high or low temperature, humidity, voltage, pressure, vibration, and/or combinations of stresses to accelerate or stimulate the failure mechanisms.

Accelerated life test stresses and stress levels should be chosen so that they accelerate the failure modes under consideration but do not introduce failure modes that would never occur under use conditions. Normally, these stress levels will fall outside the product specification limits but inside the design limits.

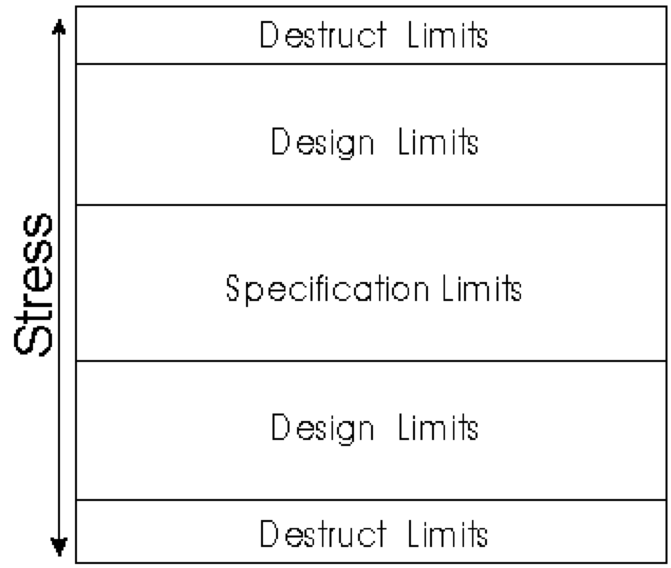


Figure 1: Typical stress range for a component, product or system.

This choice of stresses as well as stress levels and the process of setting up the experiment is of the utmost importance. Consult your design engineer(s) and material scientist(s) to determine what stimuli (stress) is appropriate as well as to identify the appropriate limits (or stress levels). If these stresses or limits are unknown, multiple tests with small sample sizes can be performed in order to ascertain the appropriate stress(es) and stress levels. The adequacy and applicability of these stresses can be confirmed through subsequent failure analysis. Information from the qualitative testing phase (i.e., torture tests, etc.) of a normal product development process can also be utilized in ascertaining the appropriate stress(es). Proper use of Design of Experiments (DOE) methodology is also crucial at this step. In addition to proper stress selection, the application of the stresses must be accomplished in some logical, controlled and quantifiable fashion. Accurate data on the stresses applied as well as the observed behavior of the test specimens must be maintained.

It is clear that as the stress used in an accelerated test becomes higher the required test duration decreases. However, as the stress moves farther away from the use conditions, the uncertainty in the extrapolation increases. This is what we jokingly refer to as the "there is no free lunch" principle. Confidence intervals provide a measure of this uncertainty in extrapolation.

3. UNDERSTANDING ACCELERATED LIFE TEST ANALYSIS

In typical life data analysis one determines, through the use of statistical distributions, a life distribution that describes the times-to-failure of a product. Statistically speaking, one wishes to determine the use level probability density function, or *pdf*, of the times-to-failure. Once this *pdf* is obtained, all other desired reliability results can be easily determined including but not limited to:

Percentage failing under warranty.
Risk assessment.
Design comparison.
Wear-out period (product performance degradation).

In typical life data analysis, this use level probability density function, or *pdf*, of the times-to-failure can be easily determined using regular times-to-failure data and an underlying distribution such as the Weibull, exponential, and lognormal distributions. In accelerated life testing analysis, however, we face the challenge of determining this use level *pdf* from accelerated life test data rather than from times-to-failure data obtained under use conditions. To accomplish this, we must develop a method that allows us to extrapolate from data collected at accelerated conditions to arrive at an estimation of use level characteristics.

3.1 *Looking at a Single Constant Stress Accelerated Life Test*

To understand the process involved with extrapolating from overstress test data to use level conditions, let's look closely at a simple accelerated life test. For simplicity we will assume that the product was tested under a single stress and at a single constant stress level. We will further assume that times-to-failure data have been obtained at this stress level. The times-to-failure at this stress level can then be easily analyzed using an underlying life distribution. A *pdf* of the times-to-failure of the product can be obtained at that single stress level using traditional approaches (for more details see [7, 10]). This overstress *pdf*, can be used to make predictions and estimates of life measures of interest at that particular stress level. The objective in an accelerated life test, however, is not to obtain predictions and estimates at the particular elevated stress level at which the units were tested, but to obtain these measures at another stress level, the use stress level. To accomplish this objective, we must devise a method to traverse the path from the overstress *pdf* to extrapolate a use level *pdf*.

The first part of Figure 2 illustrates a typical behavior of the *pdf* at the high stress (or overstress level) and the *pdf* at the use stress level. To further simplify the scenario, let's assume that a single point can describe the *pdf* for the product, at any stress level. The second part of Figure 2 illustrates such a simplification where we need to determine a way to project (or map) this single point from the high stress to the use stress.

Obviously there are infinite ways to map a particular point from the high stress level to the use stress level. We will assume that there is some road map (model or a function) that maps our point from the high stress level to the use stress level (or shows us the way). This model or function can be described mathematically and can be as simple as the equation for a line. Figure 3 demonstrates some simple models or relationships.

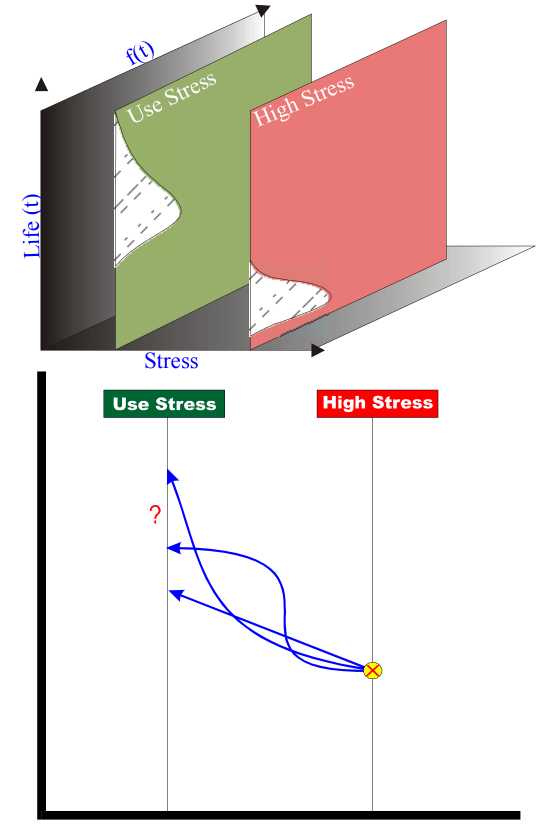


Figure 2: Traversing from a high stress to our use stress.

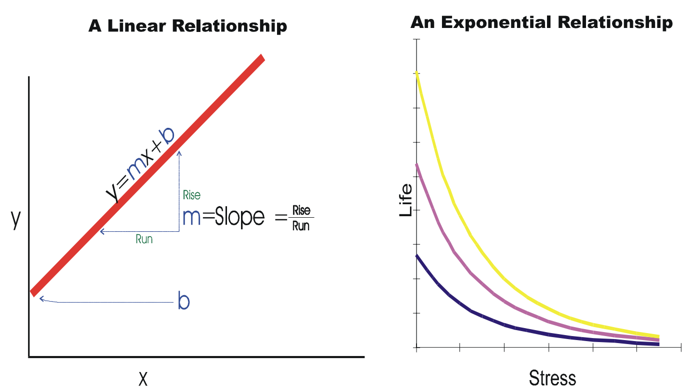


Figure 3: A simple linear and a simple exponential relationship.

Even when a model is assumed (i.e., linear, exponential, etc.), the mapping possibilities are still infinite since they depend on the parameters of the chosen model or relationship. For example, a simple linear model would generate different mappings for each slope value because we can draw an infinite number of lines through a point. If we tested specimens of our product at two different stress levels, we could begin to fit the model to the data. Obviously, the more points we have, the better off we are in correctly mapping this particular point, or fitting the model to our data. Figure 4 illustrates that you need a minimum of two stress levels to properly map the function to a use stress level.

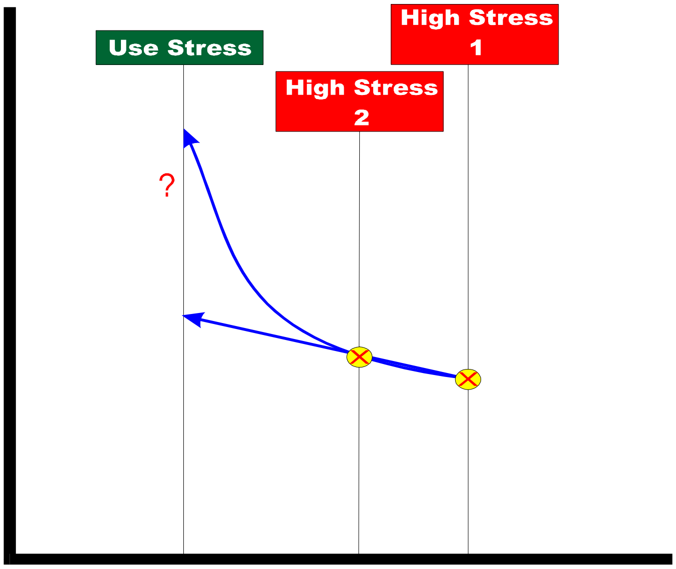


Figure 4: Testing at two (or more) higher stress levels allows us to better fit the model.

4. LIFE DISTRIBUTION AND STRESS-LIFE MODELS

Analysis of accelerated life test data, then, consists of an underlying life distribution that describes the product at different stress levels and a stress-life relationship (or model) that quantifies the manner in which the life distribution (or the life distribution characteristic under consideration) changes across different stress levels. These elements of analysis are shown graphically in Figure 5.

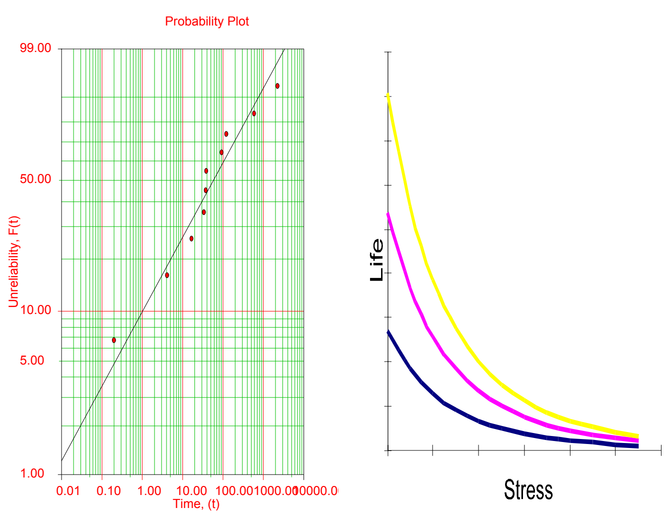


Figure 5: A life distribution and a stress-life relationship.

The combination of both an underlying life distribution and a stress-life model can be best seen in Figure 6 where a *pdf* is plotted against both time and stress.

The assumed underlying life distribution can be any life distribution. The most commonly used life distributions include the Weibull, the exponential, and the lognormal. The practitioner should be cautioned against using the exponential distribution, unless the underlying assumption of a constant failure rate can be justified. Along with the life distribution, a stress-life relationship is also used. A stress-life relationship can be one of the empirically derived relationships or a new one formulated for the particular stress and application. The data obtained from the experiment is then fitted to both the underlying life distribution and stress-life relationship.

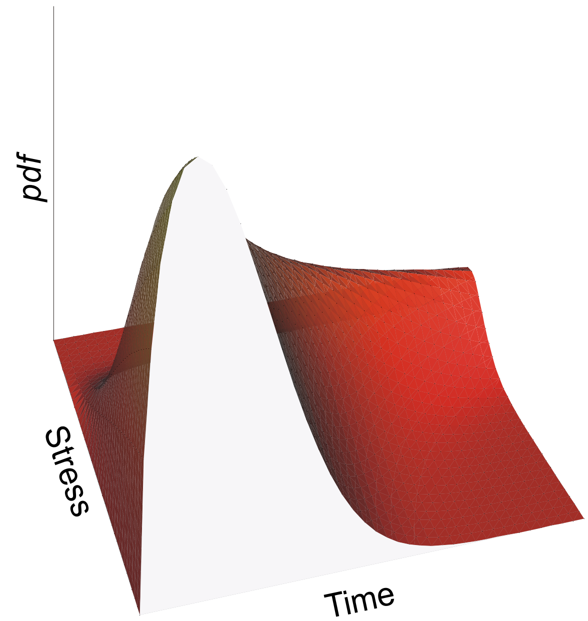


Figure 6: A three dimensional representation of the *pdf* vs. time and stress created using ReliaSoft's ALTA 1.0 software [10].

4.1 *Overview of the Analysis Steps*

With our current understanding of the principles behind accelerated life testing analysis, we will continue with a discussion of the steps involved in performing an analysis on life data that has been collected from accelerated life tests

4.1.1 *Life Distribution*

The first step in performing an accelerated life test analysis is to choose an appropriate life distribution. Although it is rarely appropriate, the exponential distribution, because of its simplicity, is very commonly used as the underlying life distribution. The Weibull and lognormal distributions, which require more involved calculations, are more appropriate for most uses. Note that the exponential distribution is a special case of the Weibull (for equal to 1).

4.1.2 *Stress-Life Relationship*

After you have selected an underlying life distribution appropriate to your data, the second step is to select (or create) a model that describes a characteristic point or a life characteristic of the distribution from one stress level to another.

The life characteristic can be any life measure such as the mean, median, etc. This life characteristic is expressed as a function of stress. Depending on the assumed underlying life distribution, different life characteristic are considered. Typical life characteristic for some distributions are shown in the next table (Table 1).

Table 1: Typical life characteristics

Distribution	Parameters	Life Characteristic
Weibull	β^{*}, η	Scale parameter, η
Exponential	λ	Mean Life ($1/\lambda$)
Lognormal	$\overline{T}$, σ^{*}	Median, $\breve{T}$

**Usually assumed constant*

For example, when considering the Weibull distribution, the scale parameter, , is chosen to be the "life characteristic" that is stress dependent, while is assumed to remain constant across different stress levels. A stress-life relationship is then assigned to .

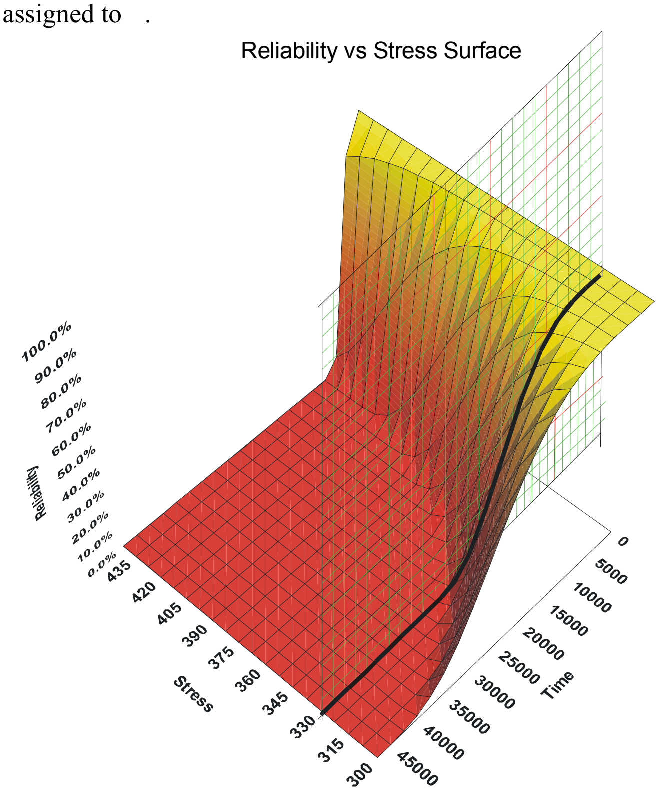


Figure 7: A graphical representation of a Weibull reliability function plotted as both a function of time and stress.

5. OVERVIEW OF SOME SIMPLE STRESS-LIFE RELATIONSHIPS

5.1 *Arrhenius Relationship*

The Arrhenius relationship is commonly used for analyzing data for which temperature is the accelerated stress. The Arrhenius model is given by,

$$L(V) = C \cdot e^{\frac{B}{V}}$$

where:

- L represents a quantifiable life measure, such as mean life, characteristic life, median life, or B(x) life, etc.
- V represents the stress level (in absolute units if it is temperature).
- C is a model parameter to be determined, (C > 0).
- B is another model parameter to be determined.

5.2 *Eyring Relationship*

The Eyring relationship is also commonly used for analyzing data for which temperature is the accelerated stress. The Eyring model is given by,

$$L(V) = \frac{1}{V} \cdot e^{-\left(A - \frac{B}{V}\right)}$$

where:

- L represents a quantifiable life measure, such as mean life, characteristic life, median life, B(x) life, etc.
- V represents the stress level.
- A is one of the model parameters to be determined.
- B is another model parameter to be determined.

5.3 *Inverse Power Law Relationship*

The inverse power law relationship (or IPL) is commonly used for analyzing data for which the accelerated stress is non-thermal in nature. The inverse power law (IPL) model is given by,

$$L(V) = \frac{1}{K \cdot V^{n}}$$

where:

- L represents a quantifiable life measure, such as mean life, characteristic life, median life, B(x) life, etc.
- V represents the stress level.
- K is a model parameter to be determined, (K > 0).
- n is another model parameter to be determined.

5.4 *Temperature-Humidity Relationship*

The temperature-humidity relationship is a two-stress relationship. It is commonly used for predicting the life at use conditions when temperature and humidity are the accelerated stresses in a test. This combination model is given by,

$$L(U,V) = A \cdot e^{\left(\frac{\phi}{V} + \frac{b}{U}\right)}$$

where:

- is one of the three parameters to be determined.
- b is the second of the three parameters to be determined (also known as the activation energy for humidity).
- A is the third of the three parameters to be determined.
- U is the relative humidity.
- V is temperature (in absolute units).

5.5 *Temperature-Non-Thermal Relationship*

The temperature-non-thermal relationship is another two-stress model. This relationship is given by,

$$L(U,V) = \frac{C}{U^{n} e^{-\frac{B}{V}}}$$

where:

- U is the non-thermal stress (e.g., voltage).
- V is the temperature (in absolute scale).
- B, C, n are the parameters to be determined.

6. PARAMETER ESTIMATION

Once you have selected an underlying life distribution and stress-life relationship model to fit your accelerated test data, the next step is to select a method by which to perform parameter estimation. Simply put, parameter estimation involves fitting a model to the data and solving for the parameters that describe that model. In our case the model is a combination of the life distribution and the stress-life relationship. The task of parameter estimation can vary from trivial (with ample data, a single constant stress, a simple distribution and a simple model) to impossible. Available methods for estimating the parameters of a model include the graphical method, the least squares method and the maximum likelihood estimation method. Computer software can be used to accomplish this task [12; 10; 11].

7. RELIABILITY INFORMATION

Once the parameters of the underlying life distribution and stress-life relationship have been estimated, a variety of reliability information about the product can be derived such as:

- Warranty time.
- The instantaneous failure rate, which indicates the number of failures occurring per unit time.
- The mean life which provides a measure of the average time of operation to failure.

8. STRESS LOADING

The discussion of accelerated life testing analysis thus far has included the assumption that the stress loads applied to units in an accelerated test have been constant with respect to time. In real life, however, different types of loads can be considered when performing an accelerated test. Accelerated life tests can be classified as constant stress, step stress, cycling stress, or random stress. These types of loads are classified according to the dependency of the stress with respect to time. There are two possible stress loading schemes, loadings in which the stress is time-independent and loadings in which the stress is time-dependent. The mathematical treatment, models and assumptions vary depending on the relationship of stress to time. This tutorial deals with time-independent stresses, the most common type of stress loading. Treatment of time-dependent stresses is complex and well beyond the scope of this tutorial. Participants interested in the analysis of data utilizing time-dependent stresses can refer to [9].

8.1 *Stress is Time-Independent (Constant Stress)*

When the stress is time-independent, the stress applied to a sample of units does not vary. In other words, if temperature is the thermal stress, each unit is tested under the same accelerated temperature, e.g., 100 C, and data are recorded. This is the type of stress load that has been discussed so far.

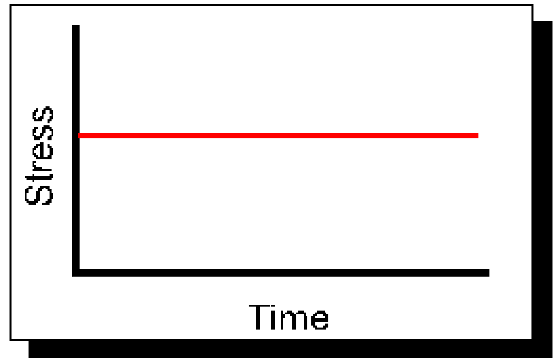


Figure 8: Graphical representation of time vs. stress in a time-independent stress loading.

This type of stress loading has many advantages over time-dependent stress loadings. Specifically:

Most products are assumed to operate at a constant stress under normal use.
It is far easier to run a constant stress test (e.g., one in which the chamber is maintained at a single temperature).
It is far easier to quantify a constant stress test.
Models for data analysis exist, are widely publicized and are empirically verified.
Extrapolation from a well executed constant stress test is more accurate than extrapolation from a time-dependent stress test.

8.2 *Stress is Time-Dependent*

When the stress is time-dependent, the product is subjected to a stress level that varies with time. Products subjected to time-dependent stress loadings will yield failures more quickly and models that fit them are thought by many to be the "holy grail" of accelerated life testing. The current state of analysis techniques for time-dependent stress loading schemes can be best expressed by a passage in Dr. Wayne Nelson's accelerated testing book [6].

Dr. Nelson writes, "Such cumulative exposure models are like the weather. Everybody talks about them, but nobody does anything about them. Many models appear in literature, few have been fitted to data and even fewer assessed for adequacy of fit. Morever, fitting such a model to data requires a sophisticated special computer program. Thus, constant stress tests are generally recommended over step-stress tests for reliability estimation."

8.3 *Stress is Quasi Time-Dependent*

The step-stress model [6] and the related ramp-stress model are typical cases of time-dependent stress tests. In these cases, the stress is quasi time-independent. This means that the stress load remains constant for a period of time and then is stepped/ramped into a different stress level where it remains constant for another time interval until it is stepped/ramped again. There are numerous variations of this concept.

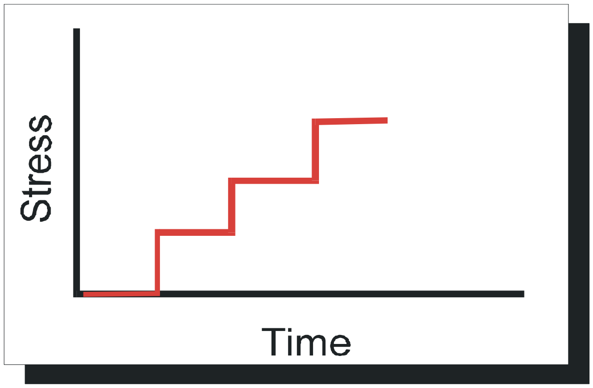


Figure 9: Graphical representation of the step-stress model.

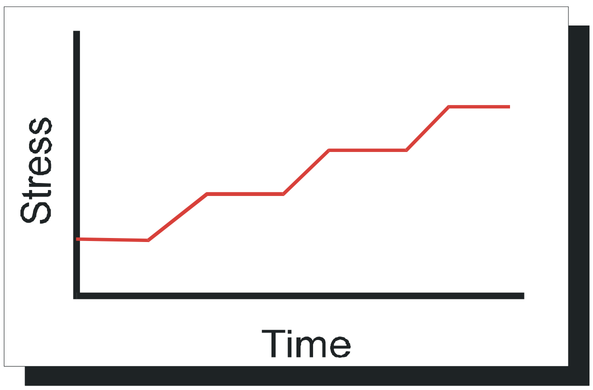


Figure 10: Graphical representation of the ramp-stress model.

8.4 *Stress is Continuously Time-Dependent*

The concept of stress-life models that includes stress as a continuous function of time has not been widely contemplated in the literature. An introduction to these models can be found in [6] and in-depth discussion and applications in [9]. Analyses of these types of stress models are more complex than the quasi time-dependent models and require advanced software packages such as [11] to accomplish.

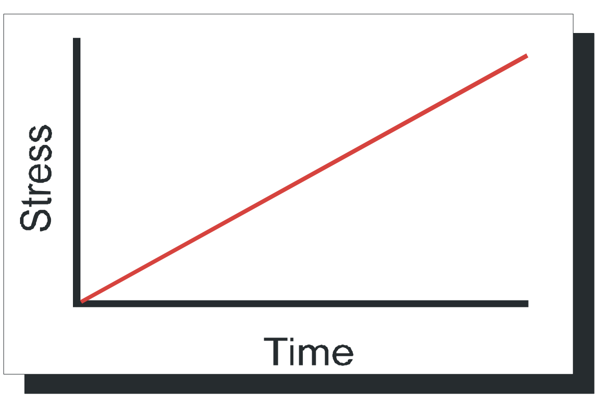


Figure 11: Graphical representation of a constantly increasing (or progressive) stress model.

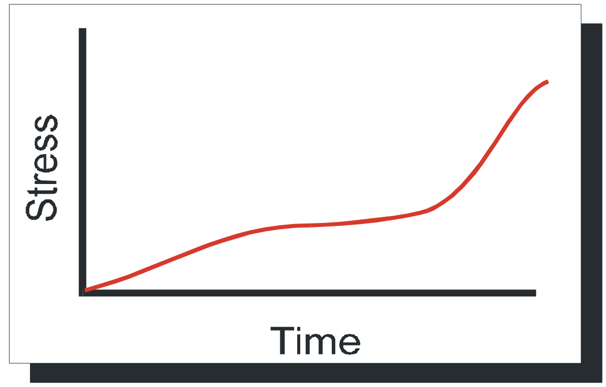


Figure 12: Graphical representation of a completely time-dependent stress model.

9. AN INTRODUCTION TO THE ARRHENIUS RELATIONSHIP

One of the most commonly used stress-life relationships is the Arrhenius. It is an exponential relationship and it was formulated by assuming that life is proportional to the inverse reaction rate of the process, thus the Arrhenius stress-life relationship is given by,

$$L(V) = C \cdot e^{\frac{B}{V}} \tag{1}$$

where:

- L represents a quantifiable life measure, such as mean life, characteristic life, median life, or B(x) life, etc.
- V represents the stress level (formulated for temperature and temperature values in absolute units i.e., degrees Kelvin or degrees Rankine. This is a requirement because the model is exponential, thus negative stress values are not possible.)
- C is one of the model parameters to be determined, ($C > 0$).
- B is another model parameter to be determined.

Since the Arrhenius is a physics-based model derived for temperature dependence, it is strongly recommended that the model be used for temperature-accelerated tests. For the same reason, temperature values must be in absolute units (Kelvin or Rankine), even though eq (1) is unitless.

The Arrhenius relationship can be linearized and plotted on a life vs. stress plot, also called the Arrhenius plot. The relationship is linearized by taking the natural logarithm of both sides in eq (1) or,

$$\ln(L(V)) = \ln(C) + \frac{B}{V} \tag{2}$$

In eq (2) $ln(c)$ is the intercept of the line and B is the slope of the line. Note that the inverse of the stress, and not the stress, is the variable. In Figure 13, life is plotted versus stress and not versus the inverse stress. This is because eq (2) was plotted on a reciprocal scale. On such a scale, the slope B appears to be negative even though it has a positive value. This is because B is actually the slope of the reciprocal of the stress and not the slope of the stress. The reciprocal of the stress is decreasing as stress is increasing $1/V$ is decreasing as V is increasing). The two different axes are shown in Figure 14.

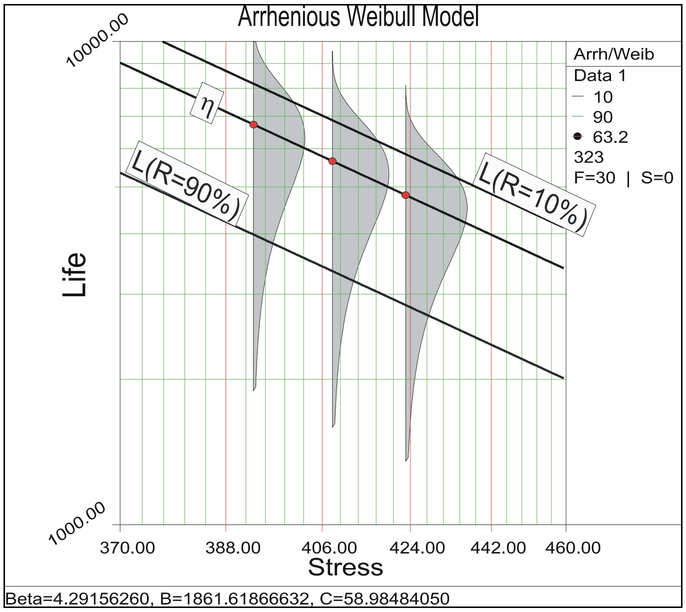


Figure 13: The Arrhenius relationship linearized on log-reciprocal paper.

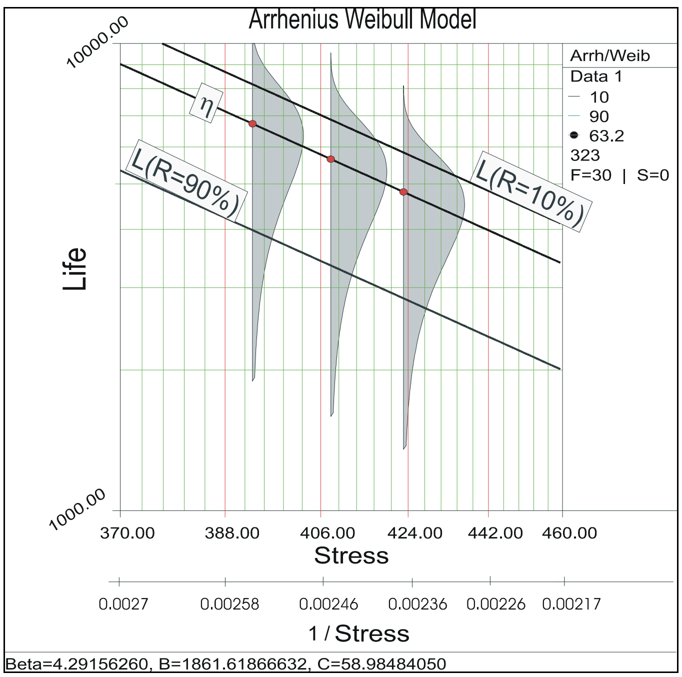


Figure 14: An illustration of both reciprocal and non-reciprocal scales for the Arrhenius relationship.

The Arrhenius relationship is plotted on a reciprocal scale for practical reasons. For example, in Figure 14 it is more convenient to locate the life corresponding to a stress level of 370K rather than to take the reciprocal of 370K (0.0027) first, and then locate the corresponding life.

The shaded areas shown in Figure 14 are the imposed *pdf*'s at each test stress level. From such imposed *pdf*'s one can see the range of the life at each test stress level, as well as the scatter in life.

9.1 *A Look at the Parameter* B

Depending on the application (and where the stress is exclusively thermal), the parameter B can be replaced by,

$$\begin{aligned} B &= \frac{E_A}{K} \\ &= \frac{\text{activation energy}}{\text{Boltzman's constant}} \\ &= \frac{\text{activation energy}}{8.623\times 10^{-5}\,\text{eV}\cdot\text{K}^{-1}} \end{aligned} \tag{3}$$

Note that in this formulation, the activation energy must be known apriori. If the activation energy is known then there is only one model parameter remaining, C. Because in most real life situations this is rarely the case, all subsequent formulations will assume that this activation energy is unknown and treat B as one of the model parameters. As it can be seen in eq (3), B has the same properties as the activation energy. In other words, B is a measure of the effect that the stress (i.e., temperature) has on the life. The larger the value of B, the higher the dependency of the life on the specific stress. Parameter B may also take negative values. In that case, life is increasing with increasing stress (see Figure 15). An example of this would be plasma filled bulbs, where low temperature is a higher stress on the bulbs than high temperature.

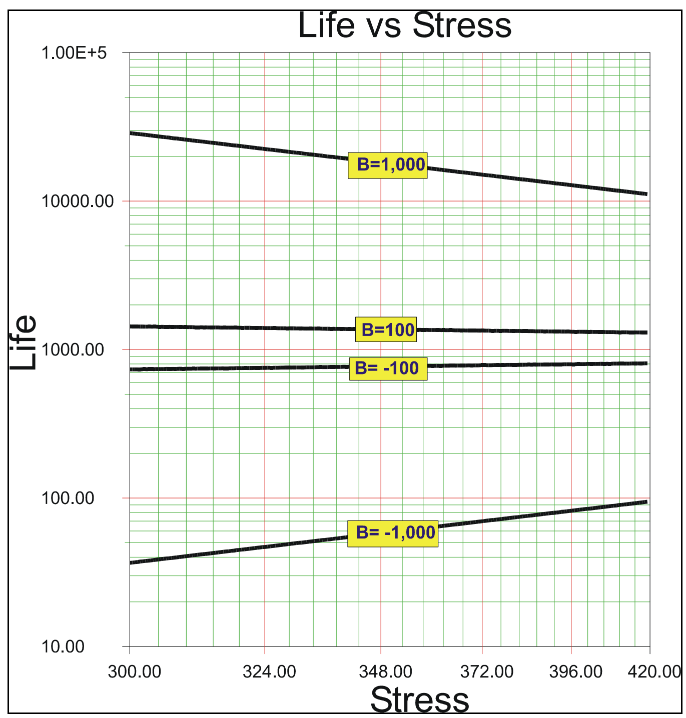


Figure 15: Behavior of the parameter B.

9.2 *Acceleration Factor*

Most practitioners use the term acceleration factor to refer to the ratio of the life (or acceleration characteristic) between the use level and a higher test stress level or,

$$A_F = \frac{L_{USE}}{L_{Accelerated}}$$

For the Arrhenius model this factor is,

$$A_F = \frac{L_{USE}}{L_{Accelerated}} = \frac{C \cdot e^{\frac{B}{V_u}}}{C \cdot e^{\frac{B}{V_A}}}$$

$$= \frac{e^{\frac{B}{V_u}}}{e^{\frac{B}{V_A}}} = e^{\left(\frac{B}{V_u} - \frac{B}{V_A}\right)}$$

Thus, if B is assumed to be known apriori (using an activation energy), the assumed activation energy alone dictates this acceleration factor!

9.3 *Arrhenius Relationship Combined with a Life Distribution*

All relationships presented must be combined with an underlying life distribution for analysis. The simplest combination is with the exponential distribution as shown next:

9.3.1 *Arrhenius Exponential*

The *pdf* of the 1-parameter exponential distribution is given by,

$$f(t) = \lambda \cdot e^{-\lambda \cdot t} \tag{4}$$

It can be easily shown that the mean life for the 1-parameter exponential distribution is given by,

$$\lambda = \frac{1}{m} \tag{5}$$

thus,

$$f(t) = \frac{1}{m} \cdot e^{-\frac{t}{m}} \tag{6}$$

The Arrhenius-exponential model *pdf* can then be obtained by setting $m = L(V)$ in eq (6). Therefore,

$$m = L(V) = C \cdot e^{\frac{B}{V}}$$

Substituting for m in eq (6) yields a *pdf* that is both a function of time and stress or,

$$f(t,V) = \frac{1}{C \cdot e^{\frac{B}{V}}} \cdot e^{-\frac{1}{C \cdot e^{\frac{B}{V}}} \cdot t}$$

Once the *pdf* is obtained all other metrics of interest (i.e., Reliability, MTTF, etc.) can be easily formulated. For more information see [12; 8].

9.3.2 *Arrhenius Weibull*

A more useful variation is the Weibull-Arrhenius formulation, which is obtained by considering the pdf for 2-parameter Weibull distribution. It is given by,

$$f(t) = \frac{\beta}{\eta} \cdot \left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}} . \tag{7}$$

The scale parameter (or characteristic life) of the Weibull distribution is . The Arrhenius-Weibull model *pdf* can then be obtained by setting $= L(V)$ in eq (7),

$$\eta = L(V) = C \cdot e^{\frac{B}{V}} , \tag{8}$$

and substituting for in eq (7),

$$f(t,V) = \frac{\beta}{C \cdot e^{\frac{B}{V}}} \cdot \left(\frac{t}{C \cdot e^{\frac{B}{V}}}\right)^{\beta-1} e^{-\left(\frac{t}{C \cdot e^{\frac{B}{V}}}\right)^{\beta}} . \tag{9}$$

An illustration of the *pdf* for different stresses is shown in Figure 16. As expected, the *pdf* at lower stress levels is more stretched to the right, with a higher scale parameter, while its shape remains the same (the shape parameter is approximately 3 in Figure 16). This behavior is observed when the parameter B of the Arrhenius model is positive. Figure 17 illustrates the behavior of the reliability function for the same parameter set.

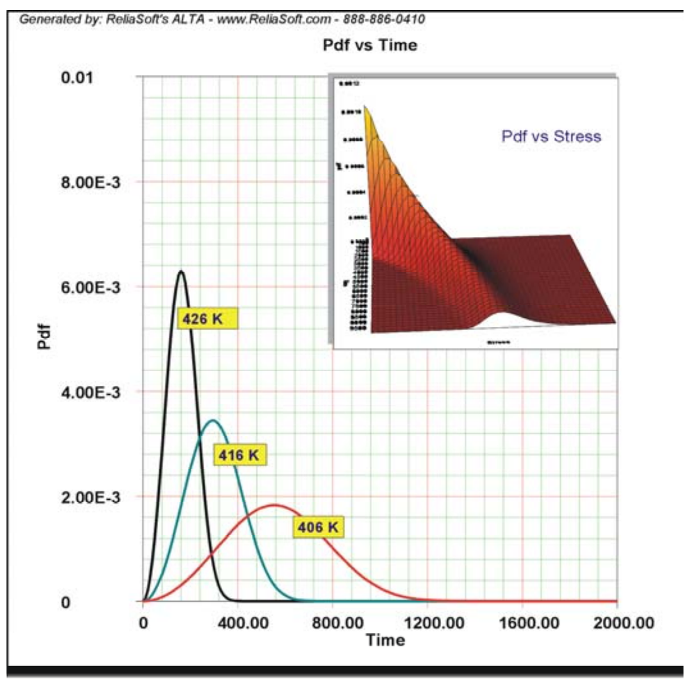


Figure 16: Behavior of the probability density function at different stresses and with the parameters held constant.

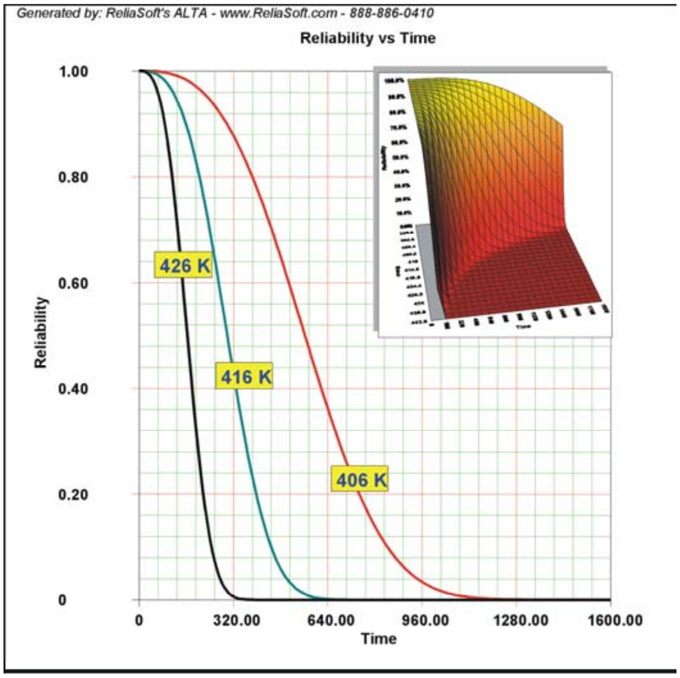


Figure 17: Behavior of the reliability function at different stresses and constant parameter values.

The advantage of using the Weibull distribution as the life distribution lies in its flexibility to assume different shapes.

9.3.3 *Example*

Consider the following times-to-failure data at three different stress levels.

Table 2: Times-to-failure data at three different stress levels.

Stress	393 K	408 K	423 K
Time Failed (hrs)	3850	3300	2750
	4340	3720	3100
	4760	4080	3400
	5320	4560	3800
	5740	4920	4100
	6160	5280	4400
	6580	5640	4700
	7140	6120	5100
	7980	6840	5700
	8960	7680	6400

The data were analyzed jointly and with a complete MLE solution over the entire data set, using [10]. The analysis yields,

$$\widehat{\beta} = 4.291$$

$$\widehat{B} = 1861.618$$

$$\widehat{C} = 58.984$$

Once the parameters of the model are estimated, extrapolation and other life measures can be directly obtained using the appropriate equations. Using the MLE method, confidence bounds for all estimates can be obtained. Note in Figure 18 below that the more distant the accelerated stress from the operating stress, the greater the uncertainty of the extrapolation. The degree of uncertainty is reflected in the confidence bounds.

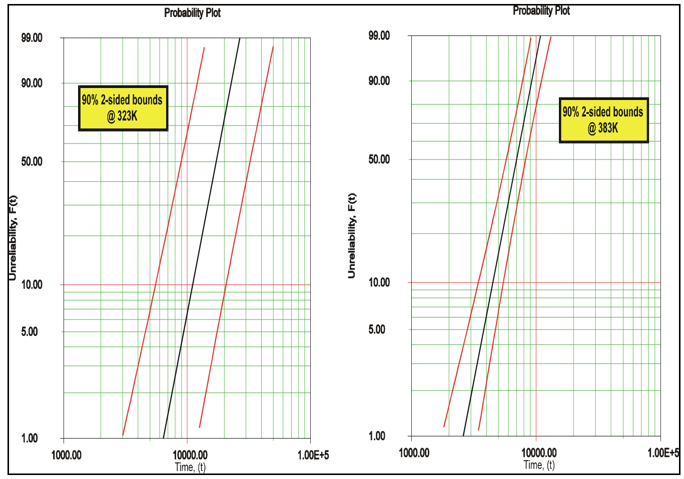


Figure 18: Comparison of the confidence bounds for different use stress levels.

9.4 *Other Single Constant Stress Models*

The same formulations can be applied to other models such as the

- Eyring relationship (exponential relationship).
- Inverse Power Law relationship (power relationship).
- Coffin Manson relationship (power relationship utilizing a ΔV for stress).

One must be cautious in selecting a model. The physical characteristics of the failure mode under consideration must be understood and the selected model must be appropriate. As an example, in cases where the failure mode is fatigue the use of an exponential relationship would be inappropriate since the physical mechanism are based on a power relation, thus a power model would be more appropriate (i.e., Inverse Power Law model).

10. AN INTRODUCTION TO TWO-STRESS MODELS

10.1 *Temperature-Humidity Relationship Introduction*

A variation of the Eyring relationship is the temperature-humidity (T-H) relationship, which has been proposed for predicting the life at use conditions when temperature and humidity are the accelerated stresses in a test. This combination model is given by,

$$L(U,V) = A \cdot e^{\left(\frac{\phi}{V} + \frac{b}{U}\right)}$$

where,

- is one of the three parameters to be determined,
- b is the second of the three parameters to be determined (also known as the activation energy for humidity),
- A is a constant and the third of the three parameters to be determined,
- U is the relative humidity (decimal or percentage),
- V is temperature (in absolute units)

Since life is now a function of two stresses, a life vs. stress plot can only be obtained by keeping one of the two stresses constant and varying the other one. In Figure 19 below, data obtained from a temperature and humidity test were analyzed and plotted on log-reciprocal paper. On the first plot, life is plotted versus temperature with relative humidity held at a fixed value. On the second plot, life is plotted versus relative humidity with temperature held at a fixed value.

Note that in Figure 19 the points shown in these plots represent the life characteristics at the test stress levels (the data were fitted to a Weibull distribution, thus the points represent the scale parameter,). For example, the points shown in the first plot represent at each of the test temperature levels (two temperature levels were considered in this test).

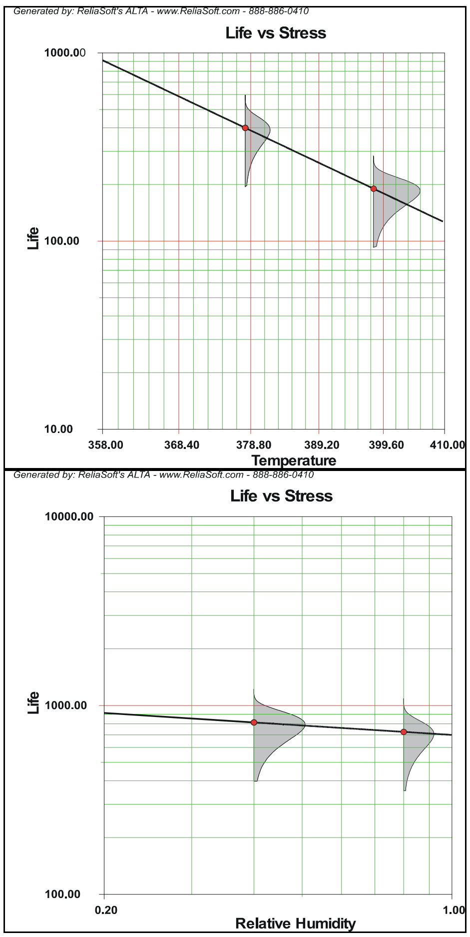


Figure 19: Life vs. stress plots for the Temperature-Humidity model, holding humidity constant on the first plot and temperature constant on the second.

10.1.1 *A Note about T-H Data*

When using the T-H relationship, the effect of both temperature and humidity on life is sought. For this reason, the test must be performed in a combination manner between the different stress levels of the two stress types. For example, assume that an accelerated test is to be performed at two temperature and two humidity levels. The two temperature levels were chosen to be 300K and 343K. The two humidity levels were chosen to be 0.6 and 0.8. It would be wrong to perform the test at (300K, 0.6) and (343K, 0.8). Doing so would not provide information about the temperature-humidity effects on life. This is because both stresses are increased at the same time and therefore it is unknown which stress is causing the acceleration on life. A possible combination that would provide information about temperature-humidity effects on life would be (300K, 0.6), (300K, 0.8) and (343K, 0.8). It is clear that by testing at (300K, 0.6) and (300K, 0.8) the effect of humidity on life can be determined (since temperature remained constant). Similarly, the effects of temperature on life can be determined by testing at (300K, 0.8) and (343K, 0.8) (since humidity remained constant).

10.1.2 *An Example Using the T-H Model*

The following data were collected after testing twelve electronic devices at different temperature and humidity conditions:

Table 3: T-H Data

Time, hr	Temperature, K	Humidity
310	378	0.4
316	378	0.4
329	378	0.4
411	378	0.4
190	378	0.8
208	378	0.8
230	378	0.8
298	378	0.8
108	398	0.4
123	398	0.4
166	398	0.4
200	398	0.4

Using [10], the following results were obtained:

$$\widehat{\beta} = 5.874$$

$$\hat{A} = 0.0000597$$

$$\widehat{b} = 0.281$$

$$\widehat{\phi} = 5630.330$$

10.2 *Temperature-Non-Thermal Relationship Introduction*

When temperature and a second non-thermal stress (e.g., voltage) are the accelerated stresses of a test, then the Arrhenius and the inverse power law models can be combined to yield the temperature-non-thermal (T-NT) model. This model is given by,

$$L(U,V) = \frac{C}{U^n e^{-\frac{B}{V}}}$$

where,

- U is the non-thermal stress (i.e., voltage, vibration, etc.),
- V is the temperature (in absolute units)
- B, C, and n are the parameters to be determined.

In Figure 20 below, data obtained from a temperature and voltage test were analyzed and plotted on a log-reciprocal scale. In the first plot, life is plotted versus temperature, with voltage held at a fixed value. In the second plot life is plotted versus voltage, with temperature held at a fixed value.

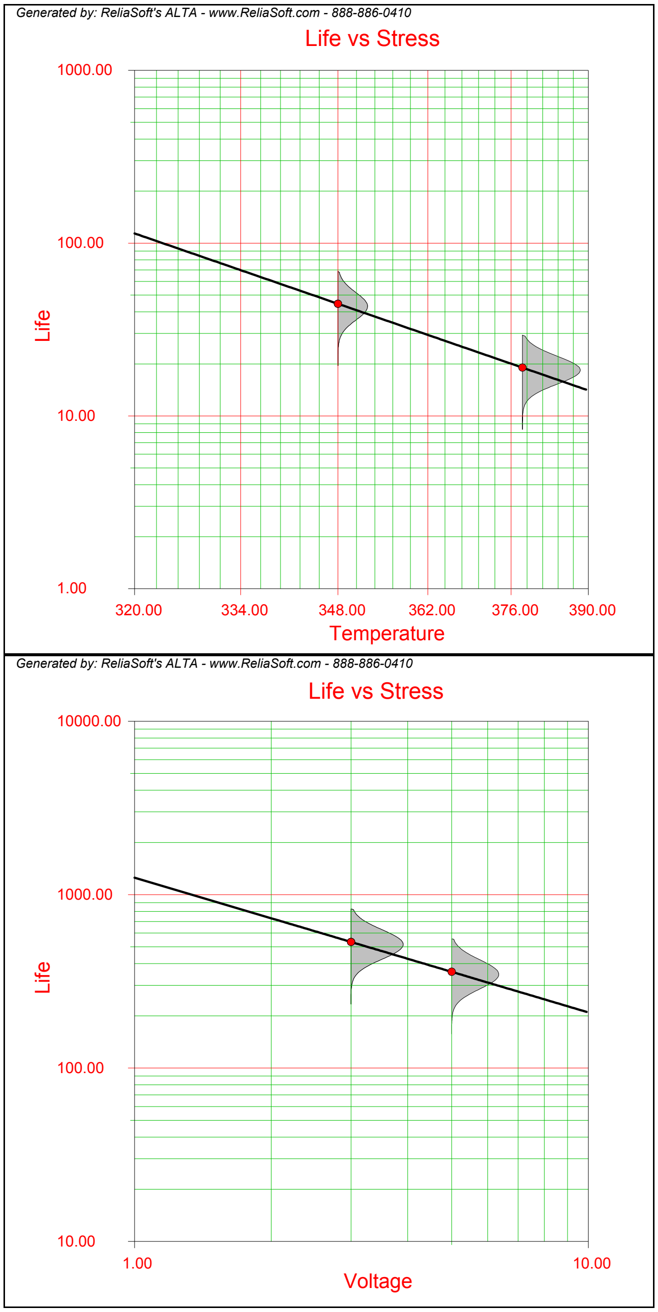
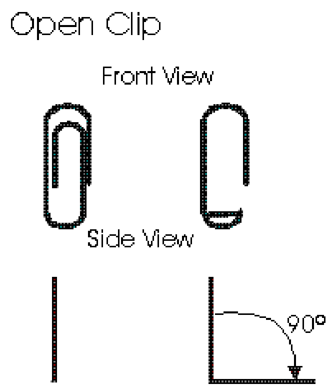


Figure 20: Life vs. stress plots for the Temperature-Humidity model, holding voltage constant on the first plot and temperature constant on the second.

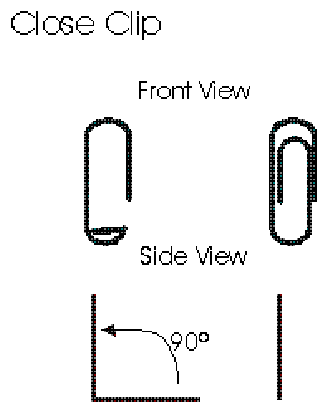
11. A VERY SIMPLE TUTORIAL EXAMPLE

To illustrate the principles behind accelerated testing, consider the following simple example that involves a paper clip and can be easily and independently performed by the reader. The objective was to determine the mean number of cycles-to-failure of a given paper clip. The use cycles were assumed to be at a 45 bend. The acceleration stress was determined to be the angle to which we bend the clips, thus two accelerated bend stresses of 90 and 180 were used. The paper clips were tested using the following procedure for the 90 bend. A similar procedure was also used for the 180 and 45 test.



1. To Open the Paper Clip.

1. With one hand, hold the clip by the longer, outer loop.
2. With the thumb and forefinger of the other hand, grasp the smaller, inner loop.
3. Pull the smaller, inner loop out and down 90 degrees so that a right angle is formed as shown.



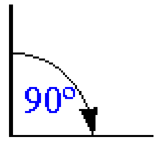
2. To Close the Paper Clip.

1. With one hand, continue to hold the clip by the longer, outer loop.
2. With the thumb and forefinger of the other hand, grasp the smaller, inner loop.
3. Push the smaller inner loop up and in 90 degrees so that the smaller loop is returned to the original upright position in line with the larger, outer loop as shown.
4. This completes one cycle.

3. Repeat until the paper clip breaks. Count and record the cycles-to-failure for each clip.

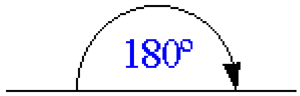
At this point the reader must note that the paper clips used in this example were "Jumbo" paper clips capable of repeated bending, different paper clips will yield different results. Additionally, and so that no other stresses are imposed, caution must be taken to assure that the rate at which the paper clips are cycled remains the same across the experiment.

For the experiment a sample of six paper clips was tested to failure at both 90 and 180 bends. A base test sample of six paper clips was tested at a 45 bend (the assumed use stress level) to confirm the analysis. The cycles-to-failure data obtained are given next.



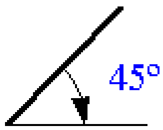
Cycles-to-failure at 90

16, 17, 18, 21, 22, 23 cycles.



Cycles-to-failure at 180

4, 5, 5, 5.5, 6, 6.5 cycles.



Cycles-to-failure at 45

58, 63, 65, 72, 78, 86 cycles.

The accelerated test data were then analyzed in [10], assuming a lognormal life distribution (fatigue) and an inverse power law relationship (non-thermal) for the stress-life model. The analysis and some of the results are shown in the next figures. The base data were analyzed using [12] and a base MTTF estimated. In this case our accelerated test correctly predicted the MTTF as verified by our base test.

It is interesting to note (see Figure 23) that mathematically one can come up with very high acceleration factors. However for one to accomplish this, these stresses must be foolishly high (i.e., 360+ degree bend on the paper clips) and would cause the product to fail under modes that are not realistic.

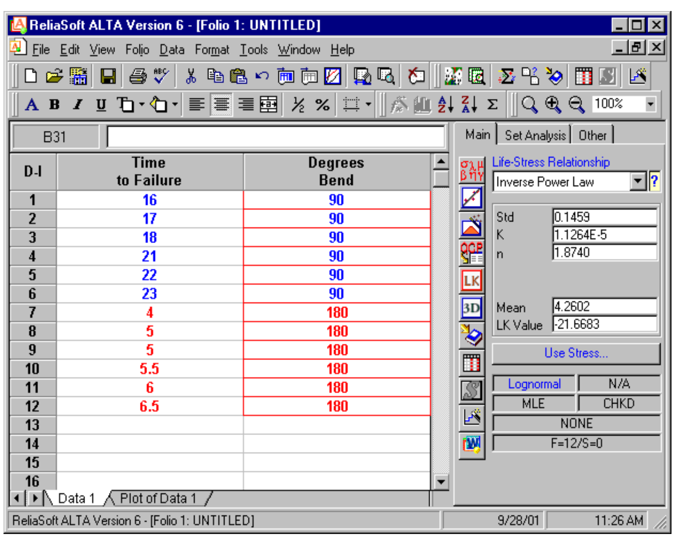


Figure 21: The accelerated test data analyzed in [10].

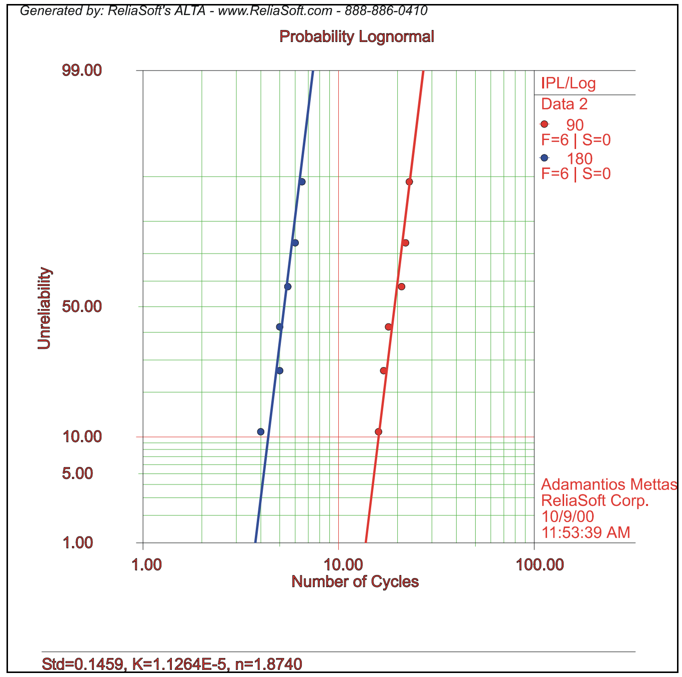


Figure 22: Resulting Probability plot for 90 and 180 bends.

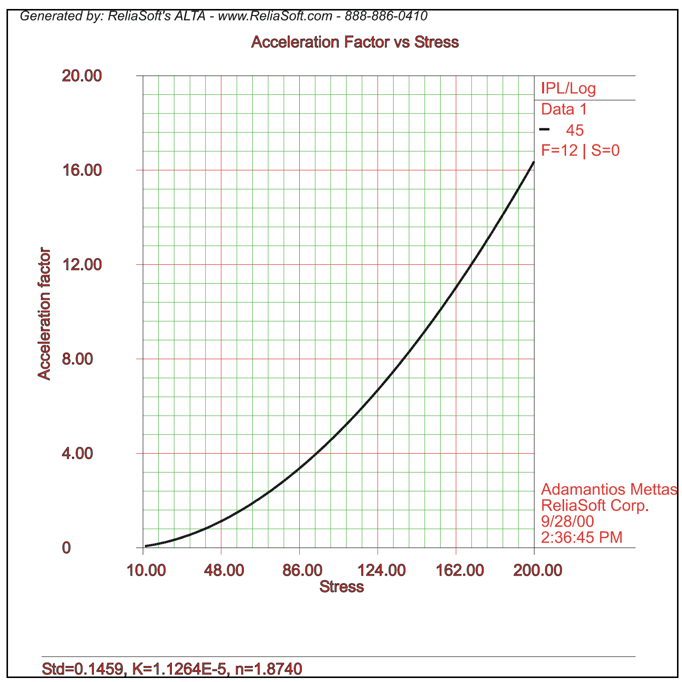


Figure 23: The resulting acceleration factor versus stress plot.

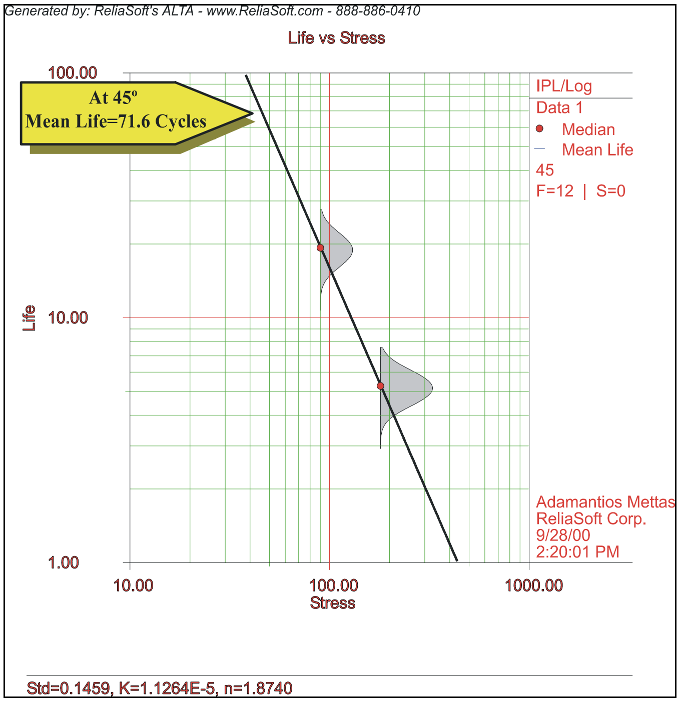


Figure 24: The resulting life versus stress plot from [10]. Note that from the plot the estimated MTTF at a 45° bend is 71.6 cycles. This was estimated utilizing the 90° and 180° bend data.

Note that the base 45 data analyzed in [12], utilizing a lognormal distribution yielded an MTTF estimate of 70.33 cycles.

12. ADVANCED CONCEPTS

12.1 *Confidence Bounds*

The confidence bounds on the parameters and a number of other quantities such as the reliability and the percentile can be obtained based on the asymptotic theory for maximum likelihood estimates, for complete and censored data. This type of confidence bounds, are most commonly referred to as the Fisher matrix bounds.

12.2 *Multivariable Relationships*

So far in this tutorial the life-stress relationships presented have been either single stress relationships or two stress relationships. In most practical applications however, life is a function of more than one or two variables (stress types). In addition, there are many applications where the life of a product as a function of stress and of some engineering variable other than stress is sought. A multivariable relationship is therefore needed in order to analyze such data.

Such a relationship is the general log-linear relationship, which describes a life characteristic as a function of a vector of n stresses. Mathematically the model is given by,

$$L(\underline{X}) = e^{\left(a_0 + \sum_{i=1}^{m} a_i X_i\right)},$$

where:

- α_j are model parameters.
- $\underline{X}$ is a vector of n stresses.

Note that a reciprocal transformation on X, or X=1/V will result to an exponential life stress relationship, while a logarithmic transformation, X=ln(V) results to a power life stress relationship.

12.3 *Time-Varying Stress Models*

When the test stresses are time-dependent (see Section 8), the life-stress relationships can be extended to account for this type of stresses. As an example consider an exponential life stress relationship utilizing a time-varying stress:

$$L(V(t)) = Ce^{\left(\frac{B}{V(t)}\right)}$$

Treatment and analysis of time-varying stresses requires further assumptions and more complex analysis techniques [6, 9, 11].

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